GENERIC PROPERTIES OF EAS GONIOMETER

Whatever definition of any property of a fixed local goniometer under study uses only average behavior of Extensive Air Showers (EAS); neither any fluctuations throughout the particle cascade development nor inaccuracy of measurements are taken into account hereinafter.

The EAS energy spectrum approximation described in the part «EAS ENERGY SPECTRUM» is used in the subsequent calculations.

The properties of a goniometer are predetermined mainly by the properties and location of the particle detectors used. The local rectangular coordinate system is used with the origin in the center of inertia of all detectors of the goniometer (with equal weights). Z-axis is directed straight up and the XY plane is disposed horizontally.

Let us assume that the goniometer contains N detectors $d=1,2,...N$, and the №d detector is located in the point $\mathbf{r}_d$ [m].[†]

The characteristic function of the installation

The principal point in the goniometer's properties definitions takes the conception of the characteristic function ${}^{(G)}\Xi$ of the installation. This is the indicator of the installation's response on the EAS front passage through the certain goniometer G location.

The characteristic function of the installation's sensitive region relative to the certain EAS is defined as:

$$^{(G)}\Xi\left(E;\ R,\alpha;\ \vartheta,\varphi\right)=\prod_{d=1}^{N}\Theta\Big(\mathrm{n}\left(E,X^{\uparrow},\mathrm{dist}(\mathbf{r}_d,R,\alpha,\vartheta,\varphi),S_d\right)-n_d^*\Big)=$$

$$=\begin{cases}\mathbf{1}: & \text{if the expected numbers of ionizing particles } \textit{in all detectors exceed} \\ & \text{the thresholds of senitivity } n_d^* \text{ of corresponding detectors;} \\ & \text{i.e. the installation } \textit{observes} \text{ the EAS} \\ \mathbf{0}: & \text{if the expected numbers of ionizing particles even in one detector is less then} \\ & \text{the corresponding threshold of senitivity } n_d^*; \\ & \text{i.e. the installation } \textit{do not observe} \text{ the EAS}\end{cases}$$

Here $\Theta(x)=\begin{cases}1, & x>0\\ 0, & x<0\end{cases}$ is the usual Heaviside step function.

The aggregate variable G *describes the fixed properties of the certain goniometer*:

$$G=\left[X^{\uparrow},\{\mathbf{rr}\},\{SS\},\{nn^*\}\right]$$

In other words the symbol G serves as a proper name of the known fixed EAS goniometer.

The listed parameters of the goniometer G stand for:

$X^{\uparrow}$ [g/cm^2] – the upright atmospheric mass depth over the installation's location;
$\{\mathbf{rr}\}$ [m] – the set of coordinates of all detectors of the installation,
$\mathbf{r}_d$ is the vector of coordinates of the detector №d;
$\{SS\}$ [m^2] – the set of areas of the detectors' slabs,
S_d is the area of the №d detector;
$\{nn^*\}$ $\left[\frac{\text{particles}}{\text{detector}}\right]$ – the set of thresholds of sensitivity of all detectors of the installation,
n_d^* is the threshold of sensitivity of the №d detector.

The quantity "*threshold of sensitivity*" n_d^* is a measure of the effective minimal number of ionizing particles traversing the detector's sensitive region that can cause the response sufficient for the installation triggering. This value depends on the detector's inherent properties as well as on the installation's environmental conditions and even on the EAS arrival direction. That is why the effective threshold of sensitivity of the detector operating as a part of the installation is a certain average quantity. It must be estimated on the ground of the results of its regular operation as a significant component of the fixed specific installation.

[†] The physical dimensions of values are shown further in [green]

Denotations of the EAS describing quantities

E [GeV] – the total energy of the EAS observed,
R [m] – the distance from the coordinate origin to the intersection point of the EAS core trajectory with the horizontal XY plane,
α [radian] – the azimuth of the last point in the XY plane,
ϑ [radian] – zenith angle of the shower's arrival direction,
φ [radian] – azimuth angle of the shower's arrival direction,

Denotations of functions

$\mathrm{n}(E, X^{\uparrow}, \mathrm{dist}(\mathbf{r}_d, R, \alpha, \vartheta, \varphi), S_d)$ [particles/detector]
– the *calculated* number of ionizing particles of the shower under consideration that traverse the №*d* detector.
$\mathrm{dist}(\mathbf{r}_d, R, \alpha, \vartheta, \varphi)$ [m] – the distance from №*d* detector to the trajectory of the shower's core.

The calculated number of ionizing particles $\mathrm{n}(\ldots)$ of the certain EAS front expected to traverse the №*d* detector is defied, mainly, by the Nishimura - Kamata - Greisen (NKG) [1,2] formula for particles density ρ_{NKG} on the shower's front surface. The quantities applied there, that is the total electron number (positrons are included) and the shower's age parameter are defined with use of the average cascade development curve $N(E, X)$ for the shower with the total energy E [GeV] at the traversed atmospheric slant mass depth X [g/cm^2]. The specific Moliere unit of multiple scattering r_M [m] is fixed for the certain installation by the air density at the goniometer's location. The muon number density [2] is taken into account, as well:

$$\mathrm{n}(E, X^{\uparrow}, \mathrm{dist}(\mathbf{r}_d, R, \alpha, \vartheta, \varphi), S_d) =$$

The detector's sensitive area projection onto the surface of the shower's front
$$= S_d \cdot \cos\vartheta \times$$
The electron/positron density by NKG
$$\times \Big\{ \rho_{\mathrm{NKG}}\big(N(E, X^{\uparrow} \cdot V(\vartheta)), s(E, X^{\uparrow} \cdot V(\vartheta)), r_M, \mathrm{dist}(\mathbf{r}_d, R, \alpha, \vartheta, \varphi)\big) \times$$
The electrons absorption by the surrounding matter
$$\times \exp\big(\lambda_1(s(E, X^{\uparrow} \cdot V(\vartheta))) \cdot X_{\mathrm{filter}}(\vartheta, \varphi) / T_{\mathrm{filter}}\big) +$$
The muon number density
$$+ \rho_{\mu}\big(N(E, X^{\uparrow} \cdot V(\vartheta)),\ \mathrm{dist}(\mathbf{r}_d, R, \alpha, \vartheta, \varphi)\big) \Big\}$$

Exactly this function for the every detector of the goniometer is used in the definition of the characteristic function ${}^{(G)}\Xi$ of the installation's sensitive region. Here the specific properties are implied:

- every detector is constructed with use of the horizontally disposed plastic scintillator slab;
- some part of the electros and positrons of the shower's front is absorbed by the substance surrounding the installation, i.e. by the so-called "filter".

Here the quantity $X_{\mathrm{filter}}(\vartheta, \varphi)$ [g/cm^2] is the mass depth of the filter absorber in the shower's arrival direction (ϑ, φ) and the T_{filter} [g/cm^2] value is the specific mass depth of one t-unit (radiation length) for the actual absorber's substance. The special function $\lambda_1(s)$ is defined in the theory of electromagnetic cascades and the shower's age parameter $s = s(E, X^{\uparrow} \cdot V(\vartheta))$ depends on the total energy E and atmospheric slant depth $X^{\uparrow} \cdot V(\vartheta)$, while the $V(\vartheta)$ function describes the dependence of atmospheric relative depth on the zenith angle of the shower's propagation line ($V(0) \equiv 1$).

The aperture function of the goniometer *G*

The certain installation's aperture ${}^{(G)}\mathrm{Ap}(E)$ is a function of the total energy E of the shower observed, defined as an integral of the corresponding characteristic function over the region accessible for the observable showers:

$$ {}^{(G)}\mathrm{Ap}(E)=\underbrace{\int_0^{2\pi}d\varphi\int_0^{\pi/2}\sin(\vartheta)\,d\vartheta}_{\substack{\text{Complete solid angle [sr]}\\ \text{of the EAS propagations}}}\ \underbrace{\int_0^{2\pi}d\alpha\int_0^{\infty}R\,dR}_{\substack{\text{Area }[\mathrm{m}^2]\text{ of the EAS}\\ \text{cores' collection}}}\ \underbrace{\left[{}^{(G)}\Xi\left(E;R,\alpha;\vartheta,\varphi\right)\right]}_{\substack{\text{Characteristic function [1]}\\ \text{of the actual goniometer}}}\ ; \qquad \left[m^2{\cdot}sr\right] \qquad (1) $$

Practically the characteristic function ${}^{(G)}\Xi$ determines the upper bound of integration of the distance R from the coordinate origin to the intersection point of the EAS core trajectory with the horizontal XY plane for every set of angular arguments' values, i.e. restricts the maximal available distance to the core of the shower with the known properties.

Let us denote these ultimate distance up to point of the EAS core intersection with the horizontal plane XY by the function ${}^{(G)}\mathrm{R}_{\mathrm{lim}}(E;\alpha;\vartheta,\varphi)$ which depends on the azimuth of the intersection point as well as on the energy and arrival direction of the shower. This is the distance at which all values of calculated number of particles in every detector achieve the corresponding threshold of sensitivity value n^*_{d}. This distance can be estimated numerically as the point of stepwise change of the characteristic function between the **0** and **1** levels and this process defines the required function ${}^{(G)}\mathrm{R}_{\mathrm{lim}}$ for subsequent use.

So, with use of the last function the expression for the aperture function is simplified by the explicit integration:

$$ {}^{(G)}\mathrm{Ap}(E)\equiv\mathrm{Ap}(E\,|\;X^{\uparrow},\{\mathbf{rr}\},\{SS\},\{nn^*\})=\int_0^{2\pi}d\varphi\int_0^{\pi/2}\sin(\vartheta)\,d\vartheta\cdot\int_0^{2\pi}d\alpha\int_0^{{}^{(G)}\mathrm{R}_{\mathrm{lim}}(E;\alpha;\vartheta,\varphi)}R\,dR\cdot[1]= $$

$$ =\frac{1}{2}\int_0^{2\pi}\left[\int_0^{2\pi}d\varphi\int_0^{\pi/2}\left({}^{(G)}\mathrm{R}_{\mathrm{lim}}(E;\alpha;\vartheta,\varphi)\right)^2\cdot\sin(\vartheta)\,d\vartheta\right]d\alpha\,; \qquad \left[m^2{\cdot}sr\right] $$

The remaining angular integrations have to be performed numerically for certain goniometer G and any required energy E of the shower.

Hereinafter the aperture function ${}^{(G)}\mathrm{Ap}(E)$ is considered as well-defined and computable for every given set of the detectors' thresholds of sensitivity $\{nn^*\}$.

The EAS observations' rate by the goniometer *G*

If the set $\{nn^*\}$ of the detectors' thresholds of sensitivity is available for the certain installation, the anticipated rate of the EAS observations can be evaluated.

The well-known EAS energy E spectrum function $\mathrm{Spectrum}_{\mathrm{EAS}}(E)$ [EAS/($m^2{\cdot}s{\cdot}GeV{\cdot}sr$)] is used for this propose (described in the «EAS ENERGY SPECTRUM» part):

$$ \mathrm{Spectrum}_{\mathrm{EAS}}(E)=J_{\mathrm{EAS}}\cdot\mathrm{f}_{\mathrm{EAS}}(E); \qquad \int_{E_{\mathrm{lb}}}^{E_{\mathrm{ub}}}\mathrm{f}_{\mathrm{EAS}}(E)\,dE=1; $$

Here J_{EAS} [EAS/($m^2{\cdot}s{\cdot}sr$)] is the total flux of the EAS-s with the energies between the minimal feasible energy E_{lb} and maximal observable energy E_{ub}, while $\mathrm{f}_{\mathrm{EAS}}(E)$ [$1/GeV$] is the probability density function.

The average rate of the EAS observations by the goniometer G with known properties is defined by the averaging of the aperture function upon the spectrum of the EAS energies:

$$ {}^{(G)}\mathrm{Rate}=\int_{E_{\mathrm{lb}}}^{E_{\mathrm{ub}}}\mathrm{Spectrum}_{\mathrm{EAS}}(E)\cdot{}^{(G)}\mathrm{Ap}(E)\,dE=\underbrace{J_{\mathrm{EAS}}}_{[\mathrm{EAS}/(m^2\cdot s\cdot sr)]}\cdot\int_{E_{\mathrm{thr}}}^{E_{\mathrm{ub}}}\underbrace{\left[{}^{(G)}\mathrm{Ap}(E)\right]}_{[m^2\cdot sr]}\cdot\underbrace{\mathrm{f}_{\mathrm{EAS}}(E)}_{GeV^{-1}}\underbrace{dE}_{GeV}; \qquad [\mathrm{EAS}/s] \qquad (2) $$

Here the minimal feasible energy E_{lb} is the spectrum parameter undependable on the installation's properties, while the specific aperture function defines the lower threshold of observable energies E_{thr} for the certain goniometer, which restricts the actual interval of integration.

Determination of the effective threshold of sensitivity of the detectors of goniometer *G* and of the corresponding threshold of the observable energies

As a matter of fact the values of the detectors' thresholds of sensitivity $\{nn^*\}$ are unavailable beforehand. Nevertheless, the expression for the EAS observations' rate (2) allows the estimation of the required average effective thresholds if all used detectors are identical and are operating in similar conditions.

Let us estimate the series of possible average rates $_{(q)}rate$, $q=1,2,\ldots Q$ of EAS observations by the goniometer G for the series of Q arbitrarily assigned sets $_{(q)}\{nn^*\}$ of supposed thresholds of sensitivity of every detector used in installation (identical for all detectors $n_d^* = {}_{(q)}n^*$, $d=1,2\ldots N$):

$$_{(q)}rate = {}^{(G)}_{(q)}\mathrm{Rate} = \mathrm{Rate}(\underbrace{X^{\uparrow}, \{\mathbf{rr}\}, \{SS\}}_{\substack{\text{Parameters}\\ \text{of actual}\\ \text{goniometer } G}},\ \underbrace{_{(q)}\{nn^*\}}_{\substack{\text{The arbitrary}\\ \text{set of identical}\\ \text{thresholds of}\\ \text{sensitivity}}}), \qquad q=1,2,\ldots Q\,.$$

With use of the interpolation polynomial $\widetilde{\mathrm{Rate}}(n^*)$ curved upon the set $\{_{(q)}n^*, {}_{(q)}rate\}$ of points in the $(n^*, rate)$ plot, the required effective thresholds of sensitivity $\{n^*_{\mathrm{eff}}\}$ of all detectors used is estimated by numerical solving of the equation:

$$\widetilde{\mathrm{Rate}}(n^*) = experimental\_rate$$

Here the $experimental\_rate$ [EAS/s] value is the practically occurred EAS observation rate at G goniometer's performance.

Now the true dependence of aperture on the EAS energy for G goniometer can be calculated at last:

$${}^{(G)}\mathrm{Ap}(E) = \mathrm{Ap}(E\,|\ X^{\uparrow}, \{\mathbf{rr}\}, \{SS\}, \{n^*_{\mathrm{eff}}\})$$

This function allows determination of the actual lower threshold of observable energies E_{thr} as the energy value corresponding with some reasonable low-scale aperture.

<u>It is considered hereinafter that the effective thresholds of sensitivity of every detector of the certain goniometer are fixed, i.e.:</u> $G = \underbrace{\left[X^{\uparrow}, \{\mathbf{rr}\}, \{SS\}, \{n^*_{\mathrm{eff}}\}\right]}_{\text{Actual goniometer } G \text{ properties}}$

The probability density function of the EAS energies observable by the goniometer *G*

Actually the aperture function ${}^{(G)}\mathrm{Ap}(E)$ of the goniometer G describes the reduction of the energy-dependent flux of all showers that is of the energy spectrum $\mathrm{Spectrum}_{\mathrm{EAS}}(E) = J_{\mathrm{EAS}} \cdot \mathrm{f}_{\mathrm{EAS}}(E)$, to the residual flux of the showers observable by the goniometer G. Therefore the distribution of the observable showers is proportional to the product $\mathrm{Spectrum}_{\mathrm{EAS}}(E) \cdot {}^{(G)}\mathrm{Ap}(E)$ and the normalized distribution is defined as

$${}^{(G)}\mathrm{f}_{\mathrm{E}}(E) = \frac{\mathrm{Spectrum}_{\mathrm{EAS}}(E) \cdot {}^{(G)}\mathrm{Ap}(E) \cdot}{\int\limits_{E_{\mathrm{thr}}}^{E_{\mathrm{ub}}} \mathrm{Spectrum}_{\mathrm{EAS}}(E') \cdot {}^{(G)}\mathrm{Ap}(E')\, dE'} = \frac{\overbrace{\mathrm{Spectrum}_{\mathrm{EAS}}(E)}^{\mathrm{EAS}/(m^2\cdot s\cdot sr\cdot GeV)} \cdot \overbrace{{}^{(G)}\mathrm{Ap}(E)}^{(m^2\cdot sr)}}{\underbrace{{}^{(G)}\mathrm{Rate}}_{s^{-1}}}; \qquad [GeV^{-1}]$$

Here the definition of the showers' observations average rate (2) is used.

This distribution is bounded below by the threshold energy E_{thr}, which is well over the lower bound energy E_{lb} of any shower emerging in the atmosphere of the Earth. Therefore any averaging upon the energies of the values measurable by the certain goniometer G must be evaluated with use of the ${}^{(G)}\mathrm{f}_{\mathrm{E}}(E)$ energy distribution of the showers' observable by this goniometer.

The probability density function of the distances between the coordinate system origin and the EAS cores' points of intersection with the XY plane for the showers observable by the goniometer *G*

Let us omit in the aperture function definition (1) the integration by the distance R between the coordinate system origin and the shower' core intersection with the horizontal plane XY, that is let us define an auxiliary function

$$^{(G)}\mathrm{Rp}(E,R) = \underbrace{R}_{[m]} \cdot \underbrace{\int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin(\vartheta)\, d\vartheta}_{\text{Complete solid angle [sr] of the EAS propagations}} \underbrace{\int_0^{2\pi} d\alpha}_{\text{Complete circle [rad]}} \underbrace{\left[{}^{(G)}\Xi(E;R,\alpha;\vartheta,\varphi) \right]}_{\text{Characteristic function [1] of the actual goniometer}} ; \qquad [m \cdot sr \cdot rad]$$

Let us define numerically (with use of the characteristic function $^{(G)}\Xi$ values at some desired values of E and R) the ultimate zenith angle $^{(G)}\theta_{\lim}(E,R,\alpha,\varphi)$ as the angle value of stepwise change of the characteristic function between the **0** and **1** levels. The last function gets the form:

$$^{(G)}\mathrm{Rp}(E,R) = R \cdot \int_0^{2\pi} d\varphi \int_0^{2\pi} d\alpha \int_0^{^{(G)}\theta_{\lim}(E,R,\alpha,\varphi)} \sin(\vartheta)\, d\vartheta \cdot [1] =$$

$$= R \cdot \int_0^{2\pi} d\varphi \int_0^{2\pi} d\alpha \left[1 - \cos\left({}^{(G)}\theta_{\lim}(E,R,\alpha,\varphi) \right) \right]; \quad [m \cdot sr \cdot rad]$$

This function is *proportional* to the required probability density function of the mentioned distances when the shower's energy is specified. Actually it goes to zero at sufficiently big distances when the showers with specified energies are unobservable. Thereby the function $^{(G)}\mathrm{R}_{\max}(E)$ of the greatest accessible distance to the shower cores is defined numerically at given energy. Moreover, the maximal distance $^{(G)}maxR = {}^{(G)}\mathrm{R}_{\max}(E_{\mathrm{ub}})$ to the ever observable shower core is definite for this goniometer G as the upper boundary of the ever observable showers energy E_{ub} in the Earth atmosphere is limited.

The required probability density function of the mentioned distances at any observable energy of the shower is defined by averaging of the $^{(G)}\mathrm{Rp}(E,R)$ function with the energy distribution $^{(G)}\mathrm{f}_{\mathrm{E}}(E)$ of the showers' observable by this goniometer:

$$^{(G)}\mathrm{Rm}(R) = \int_{E_{\mathrm{thr}}}^{E_{\mathrm{ub}}} \underbrace{\left[{}^{(G)}\mathrm{Rp}(E,R) \right]}_{[m \cdot sr \cdot rad]} \cdot \underbrace{{}^{(G)}\mathrm{f}_{\mathrm{E}}(E)}_{[GeV^{-1}]} \underbrace{dE}_{[GeV]} = \int_{E_{\mathrm{thr}}}^{E_{\mathrm{ub}}} \left[{}^{(G)}\mathrm{Rp}(E,R) \right] \cdot \frac{\mathrm{Spectrum}_{\mathrm{EAS}}(E) \cdot {}^{(G)}\mathrm{Ap}(E)}{^{G}\mathrm{Rate}} dE =$$

$$= \underbrace{\frac{J_{\mathrm{EAS}}}{^{(G)}\mathrm{Rate}}}_{\text{const } [1/(m^2 sr)]} \int_{E_{\mathrm{thr}}}^{E_{\mathrm{ub}}} \underbrace{\left[{}^{(G)}\mathrm{Rp}(E,R) \times {}^{(G)}\mathrm{Ap}(E) \right]}_{[m \cdot sr \cdot rad] \times [m^2 sr]} \cdot \underbrace{\mathrm{f}_{\mathrm{EAS}}(E)}_{[GeV^{-1}]} \underbrace{dE}_{[GeV]} ; \qquad [m \cdot sr \cdot rad]$$

This value is proportional to the number of showers of any energy with core distances R from the coordinate origin. So the normalization defines the probability density function of the distances to the cores of observable showers at this certain goniometer:

$$^{(G)}\mathrm{f}_{\mathrm{R}}(R) = \overbrace{^{(G)}\mathrm{Rm}(R)}^{[m \cdot sr \cdot rad]} \Big/ \int_0^{^{(G)}maxR} \underbrace{^{(G)}\mathrm{Rm}(R')}_{[m \cdot sr \cdot rad]} \underbrace{dR'}_{[m]}; \qquad [m^{-1}]$$

(Only showers with $R < {}^{(G)}maxR$ are observable)

All moments of $^{(G)}\mathrm{f}_{\mathrm{R}}(R)$ distribution are well defined; the lower useful moments are:

$$^{(G)}\langle R \rangle = \int_0^{^{(G)}maxR} R \cdot {}^{(G)}\mathrm{f}_{\mathrm{R}}(R)\, dR \quad [\mathrm{m}] \; ; \qquad {}^{(G)}\langle R^2 \rangle = \int_0^{^{(G)}maxR} R^2 \cdot {}^{(G)}\mathrm{f}_{\mathrm{R}}(R)\, dR \quad [\mathrm{m}^2] \qquad (3)$$

Within the assumption of azimuthal symmetry of EAS core intersection points distribution in the XY plane it becomes possible to reconstruct the two-dimensional distribution of this (x, y) points:

$$^{(G)}\mathrm{f}^{(2)}(x,y)=\frac{1}{2\pi\sqrt{x^2+y^2}}\cdot{}^{(G)}\mathrm{f}_{\mathrm{R}}\left(\sqrt{x^2+y^2}\right)\qquad[m^{-2}]$$

The average position of all EAS cores' intersections with XY plane is obviously in the goniometer's center $(x=0, y=0)$ and the covariance matrix of this two-dimensional distribution is expressed in explicit form via the second initial moment (3) of the radial distribution $^{(G)}\mathrm{f}_{\mathrm{R}}(R)$:

$$^{(G)}D=\begin{Vmatrix} ^{(G)}\langle R^2\rangle/2 & 0 \\ 0 & ^{(G)}\langle R^2\rangle/2 \end{Vmatrix},$$

that is the standard deviation along each of (x, y) dimensions is equal to $\sqrt{^{(G)}\langle R^2\rangle/2}.\quad[m]$.

«Generic Properties of EAS Goniometer»: REFERENCES

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