TECHNIQUE
OF THE EAS FRONT MOVEMENT DIRECTION ESTIMATION

The direction of Extensive Air Shower's (EAS) front movement at any small scale CR station can be estimated by means of the so-called "EAS goniometer" (**figure 1**).The latter is an installation consisting of a scintillation detector system, registering the times of EAS front charged particles' passages through the detector. This information allows estimation of the EAS movement direction by means of the relative delays between the signals.

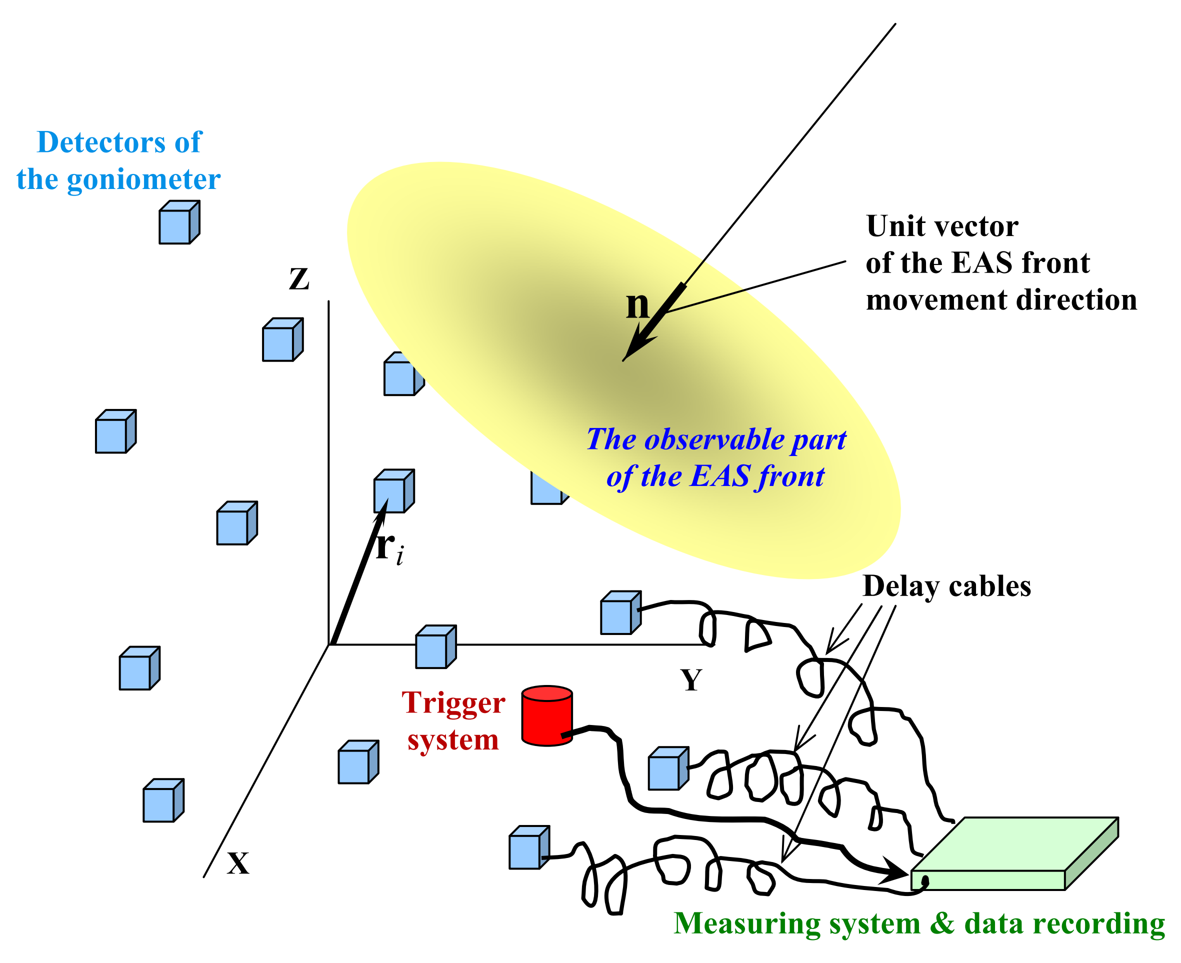

Figure 1. The concept of arrangement of most general EAS goniometer.
All signal cables from all detectors have equal length;
The rest of the delay cables conditionally are not shown

Generally the set of any number of ionizing radiation detectors, arbitrarily distributed in the 3D space, can be used as the volumetric EAS goniometer. Some "triggering structure" for EAS discovering is implied. This structure has to send a trigger signal to the measuring part of installation to fix a time reckoning of the pulses delays from EAS goniometer detectors. In particular, the EAS goniometer detectors themselves can be used for this purpose. The signals from all detectors are delayed for the common period with respect to the real moments of the pulses origins. The measuring part itself records the differences of signal arrival times with respect to the trigger signal hit time. This set of recorded numbers has to be used later (off line) to estimate the shower front movement direction.

Provided the geometric dimensions of the installation are small compared with the characteristic radius of curvature of the shower's front surface, this front surface can be approximated by a local tangent plane, moving with light velocity c. Since the plane's algebraic equation is linear with respect to all independent variables, the measured delay intervals of every detector can be expressed linearly in terms of the plane's unit direction vector components. This unit vector is perpendicular to the front's plane and shows the showers' movement direction required (**figure 1**).

The similar linear equation arises for every measured delay from every detector. So, it appears a system of linear equations with respect to the unit movement direction vector. The number of equations is equal to the number of detectors used. In the general case the detectors' number exceeds the number of vector components required. That is why the linear least-squares method has to be used for this overspecified equation system solution.

The problem solution appears as linear expressions for movement direction unit vector components in terms of all measured delay values with coefficients dependent on the coordinates of all detectors.

The goniometers for Extensive Air Showers used by the GELATICA network belong to the special case of so-called "planar goniometers". The planar goniometers (i.e. with all detectors located in a common plane) are commonly used worldwide. They have an essential shortcoming of considerable degradation of the mentioned linear least-squares method normal equation set. That is why only two components of the EAS movement direction unit vector can be estimated directly by the planar goniometer measurements.

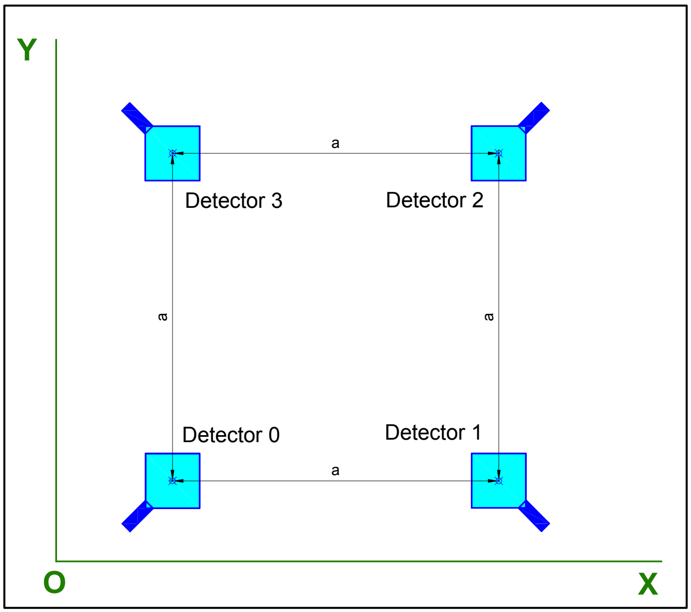

Figure 2. The basic layout of common EAS planar goniometer used by the GELATICA network

The detectors of goniometers used by the GELATICA network are commonly located in horizontal plane nearly by the corners of square with the side dimensions of a ≈ 10m (**figure 2**). This disposition is caused both by the restricted number of the DAQ's signal channels (described in the part «GELATICA ARRANGEMENT») and by the reason of constructional simplicity.

The PMT pulses, initiated by the passage of EAS charged particles through the scintillator detectors, are read by DAQ card. The equipment measures the pulse delay relative to the 4-fold pulse coincidence with $\tau = 1.25ns$ time slicing step. The data are stored on PC in the form of integer values k_0, k_1, k_2 and k_3, corresponding to the numbers of delay slices for every detector.

The spatial picture of the EAS movement direction vector estimation by some planar goniometer is shown in the **figure 3**. The EAS front plane estimated is shown in the position at the triggering moment, i.e. down the front movement direction under the plane of the detectors' location.

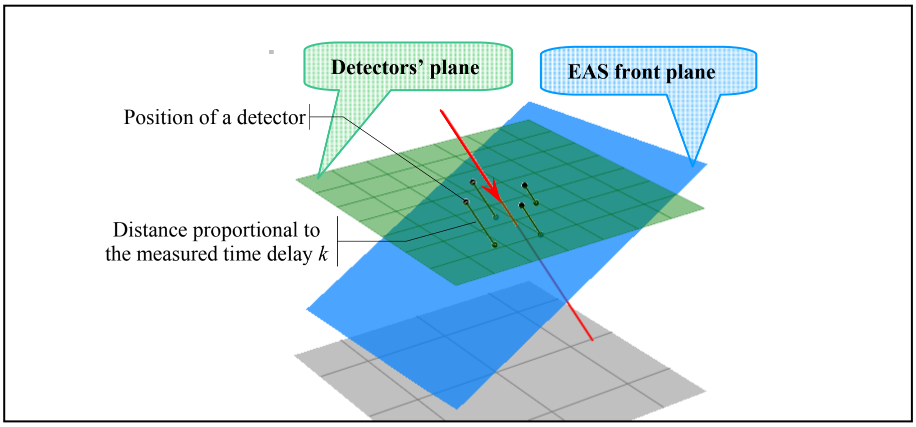

Figure 3 The spatial picture of EAS front plane movement direction estimation

For the special case of detectors' exact location in the corners of a square (**figure 2**) the estimation of the horizontal 2D projection of this direction vector **n** with respect to the XOY reference frame is

$$\left\| \begin{matrix} n_x \\ n_y \end{matrix} \right\| = \left(\frac{c \cdot \tau}{2a} \right) \cdot \left\| \begin{matrix} (k_1 + k_2) - (k_3 + k_0) \\ (k_2 + k_3) - (k_1 + k_0) \end{matrix} \right\|$$

That is in this very special symmetric case all solution coefficients (dependent on the coordinates of all detectors) are of equal absolute value. The vertical component of direction unit vector is now reconstructed by the use of unity length condition $n_z = (1 - n_x^2 - n_y^2)^{1/2}$; the dispersion of this vector horizontal components' estimation is $\sigma_x^2 = \sigma_y^2 = (c \cdot \tau / 2a)^2 \cdot [(k_0 + k_2) - (k_1 + k_3)]^2$, while the components' correlation vanishes. Here only statistical uncertainty of the front plane position is taken into account.

Practical estimations of the EAS front's direction vector **n** are evaluated with use of the known real coordinates of the detectors' location, described in the part «GONIOMETERS' GENERIC PROPERTIES». There are five independent types of fluctuations and uncertainties determining the respective covariance matrixes, which estimate the received direction vector's vagueness:

1. Covariance matrix ${}^{fr}\mathbf{D}$ is connected with fluctuations due to the limited thickness of the physical EAS front, i.e. due to the random deviations of the front particles intersection times with the real detectors relative to the conditional time of the mathematical "front plane" passage.
2. Covariance matrix ${}^{det}\mathbf{D}$ is connected with the randomness of the front particle's passage point through the detectors' areas, resulting in some random variations in the internal delays of the respective signal times.
3. Covariance matrix ${}^{loc}\mathbf{D}$ is connected with the detectors' location coordinates' measurement inaccuracy.
4. Covariance matrix ${}^{el}\mathbf{D}$ is connected with the time delays measurement inaccuracy caused by the peculiarities of the goniometer's electronic part.
5. Covariance matrix ${}^{or}\mathbf{D}$ is connected with inaccuracy in the coordinate system orientation relative to the true Geocentric Coordinate System.

The total covariance matrix is an ordinary sum of the matrixes of independent variation origins:

$${}^{tot}\mathbf{D} = {}^{fr}\mathbf{D} + {}^{det}\mathbf{D} + {}^{loc}\mathbf{D} + {}^{el}\mathbf{D} + {}^{or}\mathbf{D}$$

So, the EAS arrival direction unit vector is estimated with some error. The EAS arrival direction estimation with respect to the upper celestial hemisphere is shown in the **figure 4**. The cone of possible directions with 95% confidence level is shown too. The tipical angle eror by the GELATICA network EAS goniometers is approximately 3°. But in the horizon vicinity it grous extrimelly – because of the flatness of the detectors' disposition.

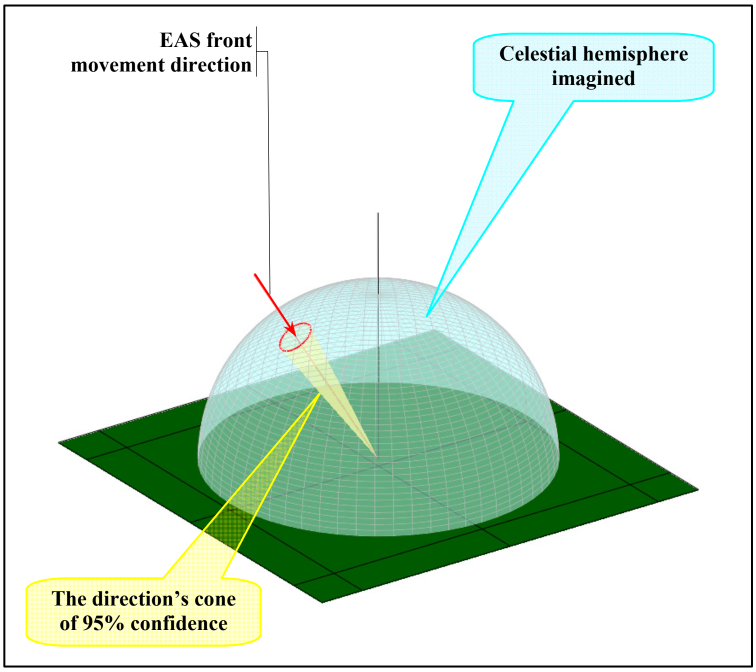

Figure 4 The EAS front movement direction estimation with respect to the upper celestial hemisphere.