THE METHOD OF ASCERTAINMENT
of the possible historical proximity of two Extensive Air Showers

As it is stated in the part «GELATICA PHYSICAL PROBLEM» some method of ascertainment whether the given pair of remote Extensive Air Showers (EAS or simply "showers") can appear the couple of the historically proximal phenomena or even the congenetic EAS couple.

At the current initial stage of the investigation one needs to establish a selection method for EAS pairs initiated by the PCR particles, possibly spatially proximal in their past. In this case the separated shower pairs can be considered as the desired congenetic EAS couples.

For this purpose the special relativistic invariant kinematic quantities are designed on the basis of available data from two EAS motion characteristics estimated by two remote goniometers. These quantities can be used to ascertain the historical proximity of the ancestors of both showers in the pair.

This method of the verifying parameters design is proposed for separation of the needed congenetic EAS couples from the whole set of the observed EAS random pairs.

Determination of the historical proximity measure of two EAS

It is essential to establish the measure of the historical proximity of two showers as *the minimal distance* ever separating the two ancestor CR particles. However, this proximity must be interpreted with an allowance for estimation error.

Let us accept the laboratory system of coordinates with fixed radius-vectors $\mathbf{r}_{01}, \mathbf{r}_{02}$ of two remote shower cores' observation. This lab system, common for both goniometers, is necessarily equivalent to the geocentric coordinate system. Let us use the universal time UTC $\hat{t}$ (common for the Earth globe) as the laboratory time. Since the gravitational effects are negligible, the lab system can be considered as the inertial one.

For the purpose of the current special problem of determination of two-shower historical proximity measure it is sufficient to consider any EAS as a free-moving point within the measurement accuracy with the region of the shower's charged-particle greatest density, i.e. with the shower's core. Let us assume that this shower-connected representing point is permanently moving along a straight line with the velocity c of light in free space. This assumption is acceptable for the CR particle with the energy sufficient for EAS origination in the Earth atmosphere since the true atmospheric track of the EAS core is almost straight and the ancestor particle is practically undeflected by the surrounding electric and magnetic fields at such energy. The motion equation of the mentioned representing point of the shower and its ancestor particle is a familiar equation of uniform motion under the accepted approximation.

The physical system under study consists of two separate showers observed by two remote goniometers at two different lab-times $\hat{t}_{01}, \hat{t}_{02}$ with the cores detected at remote points $\mathbf{r}_{01}, \mathbf{r}_{02}$ and the motion directions described by two *unit dimensionless* vectors (two "orts") $\mathbf{n}_1, \mathbf{n}_2$ of motion velocity. The ort estimation method is described in the part «GELATICA TECHNIQUE». The rare occurrences with parallel motion $(\mathbf{n}_1 = \mathbf{n}_2)$ of both showers are excluded. The available initial data from two goniometers for two observed showers are specified in the **table 1** together with the respective dispersions and covariance matrices of their estimation.

TABLE 1 INITIAL DATA

The estimated initial values	Designations
Radius-vectors of two observation points of the shower cores; the respective covariance matrices	$\mathbf{r}_{01}, \mathbf{r}_{02}$ $\mathbf{M}_1, \mathbf{M}_2$
The universal times (UTC) of those shower cores' observations; the respective dispersions	$\hat{t}_{01}, \hat{t}_{02}$ $\sigma_{t1}^2, \sigma_{t2}^2$
Orts of motion directions of two showers; the respective covariance matrices	$\mathbf{n}_1, \mathbf{n}_2$ $\mathbf{D}_1, \mathbf{D}_2$

Certainly, the coordinates of core detections may not by available for any goniometer. Though, one may substitute these data by the proper coordinates of the goniometers themselves, under the condition of sufficiently big spatial separation of installations with respect to the root-mean-square distance from the facility to the cores of the observable showers. (These values are estimated in the part «GONIOMETER'S GENERIC PROPERTIES».) The latter RMS distance can be used as the uncertainty measure of the coordinates of the shower cores.

The usual equations of uniform motion of both representing points along two skew straight lines are $\mathbf{r}_1(\hat{t}) = \mathbf{r}_{01} + c(\hat{t} - \hat{t}_{01}) \cdot \mathbf{n}_1$ and $\mathbf{r}_2(\hat{t}) = \mathbf{r}_{02} + c(\hat{t} - \hat{t}_{02}) \cdot \mathbf{n}_1$. Therefore the variable vector connecting these two moving representing points *at a time instant* $\hat{t}$ depends linearly on this time:

$$\tilde{\mathbf{\Delta}}(\hat{t}) = \mathbf{r}_2(\hat{t}) - \mathbf{r}_1(\hat{t}) = \left[(\mathbf{r}_{02} - \mathbf{r}_{01}) - c\left(\hat{t}_{02} \cdot \mathbf{n}_2 - \hat{t}_{01} \cdot \mathbf{n}_1\right) \right] + \left[c(\mathbf{n}_2 - \mathbf{n}_1) \right] \cdot \hat{t} \tag{1}$$

It is reasonable to use the special zero-time reference in the following treatment of every shower pair as a separate system. Let us take the average time $\hat{t}_c = (\hat{t}_{02} + \hat{t}_{01})/2$ of two observation times (UTC) of both showers for the system time origin. This choice helps to avoid unreasonably large numerical values in the following calculations. Thus *the system time* of any event is defined as $t = \hat{t} - \hat{t}_c$. All the times values within the given system of two showers are *the coordinate system times* hereinafter.

The showers' observation times under the system time origin are

$$t_1 = \hat{t}_{01} - \hat{t}_c = -\delta t/2; \qquad t_2 = \hat{t}_{02} - \hat{t}_c = \delta t/2; \tag{2}$$

Here the time difference $\delta t = \hat{t}_{02} - \hat{t}_{01}$ can have any sign, so the shower observation times t_1, t_2 are equal-in-magnitude and opposite in sign. The earliest observation occurs at $-|\delta t|/2$ moment, at negative system time.

Let us designate by $\delta\mathbf{r} = (\mathbf{r}_2 - \mathbf{r}_1)$ the vector connecting both points of the shower cores' observation; by $\delta\mathbf{n} = (\mathbf{n}_2 - \mathbf{n}_1)$ the difference vector of the showers' velocity orts; and by $\langle\mathbf{n}\rangle = (\mathbf{n}_1 + \mathbf{n}_2)/2$ the average vector of those orts. The last two vectors are mutually orthogonal: $(\langle\mathbf{n}\rangle^{\mathrm{T}} \cdot \delta\mathbf{n}) = 0$.[†]

The simple secondary values derived on the basis of initial data shown in the **table 1** are specified in **table 2** for referencing purpose. They are used in the following calculations.

TABLE 2 SIMPLE SECONDARY VALUES

Derived values	Definitions
Time difference of the showers' observations; the respective dispersion	$\delta t = \hat{t}_{02} - \hat{t}_{01}$ $\sigma^2_{\delta t} = \sigma^2_{t1} + \sigma^2_{t2}$
The showers' observation times; the respective dispersions	$t_1 = -\delta t/2$; $t_2 = \delta t/2$; $\sigma^2_{\delta t}/4$
The vector connecting both points of the shower cores' observation; the respective covariance matrix	$\delta\mathbf{r} = \mathbf{r}_{02} - \mathbf{r}_{01}$ $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$
The difference vector of the showers' velocity orts; the respective covariance matrix	$\delta\mathbf{n} = \mathbf{n}_2 - \mathbf{n}_1$ $\mathbf{D} = \mathbf{D}_1 + \mathbf{D}_2$
The average vector of the showers' velocity orts; the respective covariance matrix	$\langle\mathbf{n}\rangle = (\mathbf{n}_1 + \mathbf{n}_2)/2$ $(\mathbf{D}_1 + \mathbf{D}_2)/2^2 = \mathbf{D}/4$

The expression (1) for the variable vector connecting two moving representing points has simpler form when using the above definitions:

$$\tilde{\mathbf{\Delta}}(t) = \left[\delta\mathbf{r} - \langle\mathbf{n}\rangle(c\,\delta t) \right] + \delta\mathbf{n} \cdot ct \tag{3}$$

[†] The symbol $^{\mathrm{T}}$ indicates the transpose of an algebraic vector or matrix

The smallest value of this vector's length (in the whole unbounded period of the showers' representing points' motion) will be assumed as the source for design of the desired measure of the showers' historical proximity. Let us use for this propose the relativistic invariant quantity, the squared interval between the representing points at any time instance t. The spatial vector $\tilde{\mathbf{\Delta}}(t)$ connecting the two events (3) is known. The time difference between them is zero by definition. So the squared interval is $s^2(t) = -(c\cdot 0)^2 + \tilde{\mathbf{\Delta}}(t)^{\mathrm{T}} \cdot \tilde{\mathbf{\Delta}}(t)$. This function has the smallest value at the moment τ:

$$\tau = -\frac{1}{c}\cdot\frac{(\delta\mathbf{r}^{\mathrm{T}}\delta\mathbf{n})}{(\delta\mathbf{n}^{\mathrm{T}}\delta\mathbf{n})};$$

For $\tau < 0$, i.e. in the system past, the representing points have been in the nearest position. For $\tau > 0$ the representing points seem approaching each other, although in reality they are absorbed in the underlying ground.

The considered scheme of simultaneous motion of two representing points along two straight skew lines is shown in the **figure 1**. The special case of the points' closest approach *before* the earliest observation of one of the showers is shown here.

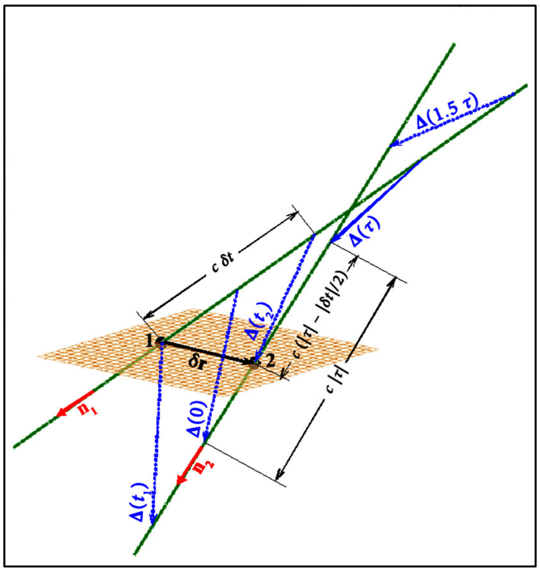

Figure 1. The scheme of two showers' representing points' motion. Two straight skew trajectories of the representing points are shown as green lines. Symbols **1** and **2** indicate the shower cores' observation points. The vector $\delta\mathbf{r}$ connects these points. The motion directing orts of both representing points are shown in red. The vectors connecting the representing points at the specified moments are shown in blue. The special vector of the points' most close approach at the moment τ is shown too, as well as lengths of the characteristic segments of trajectories. The traversed path $c\cdot(|\tau|-|\delta t|/2)$ of the №2 representing point from the moment of the points' most close approach τ till the moment t_2 of the shower observation by the №2 goniometer is also shown.

At the moment τ of the points' closest approach the variable connecting vector obtains value:

$$\tilde{\mathbf{\Delta}}(\tau) = \delta\mathbf{r} - \langle\mathbf{n}\rangle(c\,\delta t) - \frac{(\delta\mathbf{r}^{\mathrm{T}}\delta\mathbf{n})}{(\delta\mathbf{n}^{\mathrm{T}}\delta\mathbf{n})}\cdot\delta\mathbf{n},$$

so the respective relativistic invariant squared interval obtains the minimal value $\Delta^2 \equiv s^2(\tau) = \tilde{\mathbf{\Delta}}(\tau)^{\mathrm{T}}\cdot\tilde{\mathbf{\Delta}}(\tau)$. The latter is independent of the choice of the inertial coordinate system such that the dispersion of its estimate can be determined in any inertial system, including the initial laboratory system. This well-defined quantity describes the estimation precision of nonvarying value Δ^2, so it is invariant too. The dispersion $\sigma^2_{\Delta^2}$ of squared interval Δ^2 as well as the dispersion σ^2_{τ} of the moment τ of the closest approach of the representing points can be estimated using the data from the **table 2**. The respective expressions are shown in the **table 3**.

We shall use the Lorentz-invariant space-like interval $\Delta = \sqrt{\Delta^2}$ between the representing points at the moment of their closest approach as a measure of the historical proximity of the showers of the given pair. The dispersion of this quantity is equal to $\sigma^2_{\Delta} = \sigma^2_{\Delta^2}/(4\Delta^2)$.

TABLE 3 THE KINEMATIC CHARACTERISTICS OF TWO SHOWER-REPRESENTING POINTS' APPROACH AND RESPECTIVE DISPERSIONS

System describing values		Defining expressions
Coordinate time of the closest approach		$\tau = -\frac{1}{c}\cdot\frac{(\delta\mathbf{r}^{\mathrm{T}}\delta\mathbf{n})}{(\delta\mathbf{n}^{\mathrm{T}}\delta\mathbf{n})}$
Dispersion of time of the closest approach		$\sigma_\tau^2 = \frac{1}{c^2(\delta\mathbf{n}^{\mathrm{T}}\delta\mathbf{n})^2}\left\{\begin{matrix}(\delta\mathbf{n}^{\mathrm{T}}\mathbf{M}\delta\mathbf{n}) + (\delta\mathbf{n}^{\mathrm{T}}\delta\mathbf{n})^2(\delta\mathbf{r}^{\mathrm{T}}\mathbf{D}\delta\mathbf{r}) + \\ +4(\delta\mathbf{r}^{\mathrm{T}}\delta\mathbf{n})\cdot[(\delta\mathbf{r}^{\mathrm{T}}\delta\mathbf{n})(\delta\mathbf{n}^{\mathrm{T}}\mathbf{D}\delta\mathbf{n}) - (\delta\mathbf{n}^{\mathrm{T}}\delta\mathbf{n})(\delta\mathbf{n}^{\mathrm{T}}\mathbf{D}\delta\mathbf{r})]\end{matrix}\right\}$
Auxiliary quantities	Closest approach vector	$\mathbf{\Delta} = \delta\mathbf{r} - (c\,\delta t)\cdot\langle\mathbf{n}\rangle - \frac{(\delta\mathbf{r}^{\mathrm{T}}\delta\mathbf{n})}{(\delta\mathbf{n}^{\mathrm{T}}\delta\mathbf{n})}\cdot\delta\mathbf{n}$
	Projection matrix $\mathbf{\Pi}$	$\Pi_\nu^\mu = \left[\delta_\nu^\mu\right] - \frac{\left[\delta n_\nu\,\delta n^\mu\right]}{(\delta n_\alpha\,\delta n^\alpha)};\quad \mu,\nu,\ldots = x,y,z$
	Special matrix $\mathbf{G}$	$\mathrm{G}_\nu^\mu = \left\{\frac{\left[\delta r_\nu\,\delta n^\mu\right]}{(\delta n_\alpha\,\delta n^\alpha)} + \frac{(\delta r_\beta\,\delta n^\beta)}{(\delta n_\alpha\,\delta n^\alpha)}\left[\delta_\nu^\mu - 2\frac{\left[\delta n^\mu\,\delta n_\nu\right]}{(\delta n_\alpha\,\delta n^\alpha)}\right]\right\}$
Squared interval of the closest approach		$\Delta^2 = \mathbf{\Delta}^{\mathrm{T}}\cdot\mathbf{\Delta}$
Dispersion of the squared interval		$\sigma_{\Delta^2}^2 = 4(\mathbf{\Delta}^{\mathrm{T}}\langle\mathbf{n}\rangle)^2\cdot c^2\sigma_{\delta t}^2 + \mathbf{\Delta}^{\mathrm{T}}\left[4(\mathbf{\Pi}^{\mathrm{T}}\cdot\mathbf{M}\cdot\mathbf{\Pi} + \mathbf{G}^{\mathrm{T}}\cdot\mathbf{D}\cdot\mathbf{G}) + (c\,\delta t)^2\cdot\mathbf{D}\right]\mathbf{\Delta}$

* The repeating indexes imply summation upon the spatial coordinates *x, y, z*.

Application of the obtained quantities to the problem of identification of shower couples with possibly congenetic ancestors

The special pairs of showers, the ancestors of which may be suspected of having common origin, must obviously possess the following properties:

- The closest approach between the corresponding representing points takes place *before* the first observation of the shower by one of the goniometers, i.e. $\tau < \min(t_1, t_2) < 0$
- The space-like interval Δ between both representing points at the moment of their closest approach coincides with zero distance within the estimation precision, i.e. $\Delta < \sigma_\Delta$.

Let us define the respective dimensionless verifying parameters distinguishing the desired couples of related showers among the whole set of the observed pairs.

It is convenient to match the time periods between the specific events in the given system of two showers' representing points with the period proper for this system, i.e. with the propagation time of electromagnetic pulse between the shower cores' observation points, i.e. $|\delta\mathbf{r}|/c$.

The earliest observation time of one of the shower cores $-|\delta t|/2$ is negative in accordance with the definition (2). Consequently, the time sequence order of two system events, exactly the earliest observation time of one of the shower cores and the moment of the closest approach of corresponding representing points, can be described by the dimensionless ratio:

$$\frac{\tau - \min(t_1,t_2)}{|\delta\mathbf{r}|/c} = \frac{c(\tau + |\delta t|/2)}{|\delta\mathbf{r}|} \tag{4}$$

This quantity is a ratio of two periods, related to the single physical system of the showers' pair, and measured in the fixed lab coordinate system. Both time periods are multiplied by the same gamma-factor under transformation to new inertial coordinate system, so the ratio (4) is the dimensionless relativistic invariant.

Unfortunately, the immediate use of the latter quantity is restricted by the technical difficulties arising from possibilities of large numerical values of either sign, which complicates pictorial presentation of all time sequencing possibilities. The usual approach of taking the logarithm of a large number can not be adopted in this case due to possible negative values; rather the well-known function of inverse hyperbolic sine can be used instead. At large values of argument this function behaves asymptotically as the logarithmic one, preserving the argument sign, and is nearly linear for small values of argument. So let us define the *time **S**equencing* verifying dimensionless parameter S for two events under consideration in the showers' system by the expression:

$$S = \operatorname{arsinh}\left(\frac{c\left(\tau + |\delta t|/2\right)}{|\delta \mathbf{r}|}\right); \qquad -\infty < S < +\infty .$$

The verifying parameter S as well as the periods' ratio (4) takes on a negative value for those special shower pairs, for which the moment of closest approach of the representing points occurs *before* the earliest observation of either of the two showers. Only the shower pair with negative verifying parameter S value can potentially be the historically proximal couple. The positive value of parameter S means that the representing points approach to each other after the earliest observation of either of the two showers. (To be more exact, they would have a chance for mutual approach in future if they were not absorbed in the Earth under the goniometers.)

Let us describe the extent of the historic proximity of the representing points of the showers by the ratio Δ/σ_Δ . If the ratio is less then unity (i.e. the invariant interval value not exceeds the limit of one standard deviation uncertainty), then the invariant distance between the representing points at the moment of the closest approach is effectively zero within the estimation precision. The shower pairs with the ratio not exceeding the unity can be considered as likely historically proximal couples. So long as this nonnegative ratio can attain large values let us define the *historical **P**roximity* verifying dimensionless parameter P by the expression:

$$P = \ln\left(\frac{\sigma_\Delta}{\Delta}\right); \qquad -\infty < P < +\infty .$$

This parameter is relativistic invariant quantity as it is a function of two Lorentz-invariants. It is positive for the historically proximal showers, and the larger the value, the more reliable is the statement of the historical proximity of both showers.

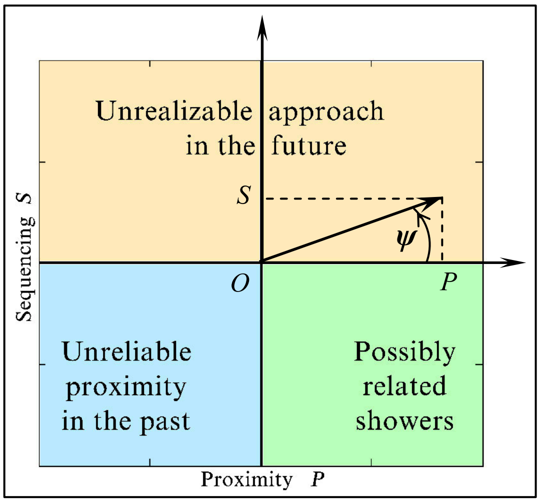

Figure 2. The plane of verifying invariant parameters for the pair of extensive air showers

The whole plane of two verifying dimensionless parameters $(P,\ S)$ can be divided into three zones as is shown in **figure 2.** The points in those zones are related:

1. With pairs of showers described by representing points which can approach one another in the future, but have to be absorbed in the ground: $S > 0$;
2. With pairs of showers described by representing points which have been in the near positions in their past, but the estimation precision of the approach invariant interval prevents the decision of true historical proximity: $S < 0,\ P < 0$;
3. With pairs of showers described by representing points which closely approach one another in their past and originate from a single point within the estimation precision: $S < 0,\ P > 0$.

Every point in the verifying parameters' plane defines the angle $\psi = \arctan(S/P)$ (**figure 2**) between the OP axis and the direction from the origin O to the given $(P,\ S)$ point. With this angle we can define united verifying criterion via $K = (2/\pi)\cdot \operatorname{arctg}(S/P); \qquad 0 \le K < 4$.

Depending on the value of criterion K the given $(P,\ S)$ point belongs to:

$0 \le K < 2$ – Zone 1: unrealizable approach in the future;

$2 \le K < 3$ – Zone 2: unreliable proximity in the past;

$3 \le K < 4$ – Zone 3: possibly historically related showers.

So the united relativistic invariant criterion K allows unambiguous definition of the admissibility of assumption that the given pair of showers can have the historically proximal ancestors. The integer bounds of the possible K-zones are illustrative, delimiting the quadrants of the verifying parameters' plane.